

# Solar neutrino oscillations and bounds on neutrino magnetic moment and solar magnetic field

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## Abstract

If the observed deficit of solar neutrinos is due to neutrino oscillations, neutrino conversions caused by the interaction of their transition magnetic moments with the solar magnetic field (spin-flavour precession) can still be present at a subdominant level. In that case, the combined action of neutrino oscillations and spin-flavour precession can lead to a small but observable flux of electron antineutrinos coming from the sun. Non-observation of these  $\bar{\nu}_e$ 's could set limits on neutrino transition moment  $\mu$  and the strength and coordinate dependence of the solar magnetic field  $B_\perp$ . The sensitivity of the  $\bar{\nu}_e$  flux to the product  $\mu B_\perp$  is strongest in the case of the vacuum oscillation (VO) solution of the solar neutrino problem; in the case of the LOW solution, it is weaker, and it is the weakest for the LMA solution. For different solutions, different characteristics of the solar magnetic field  $B_\perp(r)$  are probed: for VO solution, the  $\bar{\nu}_e$  flux is determined by the integral of  $B_\perp(r)$  over the solar convective zone, for LOW solution it depends on the derivative of  $B_\perp(r)$  at the surface of the sun, and for LMA it is determined by the magnitude of  $B_\perp$  in the neutrino production region.

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# 1 Introduction

The observed deficit of solar neutrinos [1] compared to the expectations based on the standard solar model [2] and the standard electroweak model [3] is now firmly established to be due to non-standard neutrino properties. In particular, the SNO Collaboration has demonstrated [4] that a significant fraction of solar  $\nu_e$  is converted into some other active neutrino species, which can be  $\nu_\mu$ ,  $\nu_\tau$ ,  $\bar{\nu}_\mu$  or  $\bar{\nu}_\tau$ . The most plausible and widely accepted explanation of the observed solar neutrino deficit are neutrino oscillations; however, some alternative possibilities are not ruled out yet. One of them is neutrino spin-flavour precession [5] due to the interaction of neutrino transition (flavour off-diagonal) magnetic moments with the solar magnetic field. Unlike the ordinary neutrino spin precession [6], the spin-flavour precession (SFP) can take place even if neutrinos are Majorana particles; in this case it converts left-handed  $\nu_e$  into right-handed  $\bar{\nu}_\mu$  or  $\bar{\nu}_\tau$ , which would be in accord with the SNO findings. Neutrino SFP can be resonantly enhanced in matter [7, 8], very much similarly to the resonance amplification of neutrino oscillations, the MSW effect [9].

SFP of solar neutrinos, both resonance and non-resonance, can very well account for the observed solar neutrino deficit. It yields an excellent fit of all currently available solar neutrino data (see, e.g., [10 – 18] for recent analyses), even somewhat better than that of the large mixing angle (LMA) oscillation solution, which is the best one among the oscillation solutions. However, to account for the solar neutrino data, SFP requires relatively large values of the neutrino transition magnetic moment,  $\mu \sim 10^{-11} \mu_B$  for peak values of the solar magnetic field  $B_0 \sim 100$  kG. Although such values of  $\mu$  are not experimentally excluded, they are hard to achieve in the simplest extensions of the standard electroweak model.

In the present paper we shall be assuming that the neutrino transition magnetic moment and/or solar magnetic field strength are significantly below the values necessary for the SFP mechanism to account for the solar neutrino deficiency (though not completely negligible). Our assumption is that it is neutrino oscillations that solve the solar neutrino problem, while the SFP is present at a subdominant level. What can then the observable effects of SFP be? Its influence on the survival probability of solar  $\nu_e$  will be small and essentially indistinguishable from a small change of the neutrino oscillation parameters. However, the combined action of neutrino oscillations and SFP can lead to a qualitatively new effect which is absent when only oscillations or only SFP are operative – the production of a flux of electron antineutrinos [7, 19, 20, 21, 22]. Since all the currently favoured oscillation solutions of the solar neutrino problem – LMA and LOW MSW solutions and vacuum oscillations (VO) – require the solar neutrino oscillations to be driven by a large mixing angle [23], an observable flux of solar  $\bar{\nu}_e$  can in principle be produced.

In the present paper we address the question of what can be learned about the neutrino transition magnetic moments  $\mu$  and the solar magnetic field by studying the solar  $\bar{\nu}_e$ 's. In particular, we discuss the bounds on  $\mu$  and the strength and coordinate dependence of the solar magnetic field that can be derived from the current upper limits on the solar  $\bar{\nu}_e$  flux as well as from future experiments in case the flux of  $\bar{\nu}_e$  from the sun is not observed.

Experimentally,  $\bar{\nu}_e$ 's have a very clear signature and can be easily distinguished from the other neutrino species. The main problem with detecting  $\bar{\nu}_e$ 's from the sun is the background of electron antineutrinos from nuclear reactors. This background is a steeply decreasing function of neutrino energy; it becomes negligible for  $E > (5 - 8)$  MeV. Therefore only the solar  ${}^8\text{B}$  neutrinos can contribute to an observable flux of solar  $\bar{\nu}_e$ 's and the energy interval to be studied is  $E \simeq (5 - 15)$  MeV.

Neutrino magnetic moments can also manifest themselves through the additional contribution to the  $\nu e$  scattering cross section in the solar neutrino detectors (see, e.g., [24] for recent discussions). However, these contributions can only be noticeable if  $\mu \gtrsim 10^{-10} \mu_B$ . While such large values of  $\mu$  are consistent with the current laboratory upper bounds [25], they exceed the astrophysical bounds  $\mu < (1 - 3) \times 10^{-12} \mu_B$  [26] by more than an order of magnitude. In our study we shall be assuming the astrophysical upper bounds to be satisfied and so shall neglect the effects of neutrino magnetic moment on neutrino detection.

## 2 Probability of $\bar{\nu}_e$ production

We shall assume neutrinos to be Majorana particles and consider their evolution under the combined action of oscillations and SFP. For simplicity, we shall discuss the case of just two neutrino flavours,  $\nu_e$  and  $\nu_\mu$ , and their antineutrinos.

There are essentially two ways in which  $\bar{\nu}_e$ 's can be produced: (1) the originally produced solar  $\nu_e$  first oscillate into  $\nu_\mu$ , which are then converted into  $\bar{\nu}_e$  by SFP; (2) solar  $\nu_e$  first undergo SFP and get converted into  $\bar{\nu}_\mu$ , which then oscillate into  $\bar{\nu}_e$ . This can be schematically shown as

$$\nu_{eL} \xrightarrow{\text{osc.}} \nu_{\mu L} \xrightarrow{\text{SFP}} \bar{\nu}_{eR}, \quad (1)$$

$$\nu_{eL} \xrightarrow{\text{SFP}} \bar{\nu}_{\mu R} \xrightarrow{\text{osc.}} \bar{\nu}_{eR}. \quad (2)$$

The oscillations and SFP in these two chains of conversions can either take place in the same spatial region, or be spatially separated. In the former case, the amplitudes of the processes (1) and (2) interfere. It was shown in [19] that the interference is destructive, leading to a significant suppression of the solar  $\bar{\nu}_e$  flux even if the probability of SFP is large. The reason for this is CPT invariance from which it follows that the matrix of the Majorana-type transition magnetic moments is antisymmetric; this, in turn, implies that the amplitudes (1) and (2) are of opposite sign [21]. The cancellation between the amplitudes (1) and (2) is exact when the corresponding intermediate states ( $\nu_\mu$  and  $\bar{\nu}_\mu$ ) are degenerate, e.g., in vacuum; the degeneracy is lifted by matter and/or twisting magnetic field (i.e. by a magnetic field whose direction in the plane transverse to the neutrino momentum changes along the neutrino path) [21], so inside the sun the  $\bar{\nu}_e$  production is not completely blocked. Still, it is strongly suppressed <sup>1</sup>, and so we shall concentrate on the  $\bar{\nu}_e$  production in the

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<sup>1</sup>Except, possibly, for the LOW solution of the solar neutrino problem, see discussion in Sec. 3.2.

sequence of two spatially separated processes corresponding to eq. (2): first, SFP inside the sun converts solar  $\nu_{eL}$  into  $\bar{\nu}_{\mu R}$  which then oscillate into  $\bar{\nu}_{eR}$  in vacuum on their way to the earth. The alternative possibility, corresponding to eq. (1), can be disregarded as the magnetic field in the region between the sun and the earth is negligibly small. The probability that a  $\nu_{eL}$  born inside the sun will reach the earth as  $\bar{\nu}_{eR}$  is then

$$\begin{aligned} P(\nu_{eL} \rightarrow \bar{\nu}_{eR}) &= P(\nu_{eL} \rightarrow \bar{\nu}_{\mu R}; R_\odot) \cdot P(\bar{\nu}_{\mu R} \rightarrow \bar{\nu}_{eR}; R_{es}) \\ &= P(\nu_{eL} \rightarrow \bar{\nu}_{\mu R}; R_\odot) \cdot \sin^2 2\theta \sin^2 \left( \frac{\Delta m^2}{4E} R_{es} \right), \end{aligned} \quad (3)$$

where  $R_\odot$  is the solar radius,  $R_{es}$  is the distance between the sun and the earth,  $\Delta m^2$  and  $\theta$  are the neutrino mass squared difference and mixing angle, and  $E$  is neutrino energy. We shall now concentrate on the calculation of the SFP probability  $P(\nu_{eL} \rightarrow \bar{\nu}_{\mu R}; R_\odot)$ .

The evolution of the neutrino system under the consideration is described by the following system of equations [7, 19, 21]:

$$i \nu'_{eL} = (V_e - c_2 \delta) \nu_{eL} + s_2 \delta \nu_{\mu L} + \mu B_\perp e^{i\phi} \bar{\nu}_{\mu R}, \quad (4)$$

$$i \bar{\nu}'_{eR} = -(V_e + c_2 \delta) \bar{\nu}_{eR} - \mu B_\perp e^{-i\phi} \nu_{\mu L} + s_2 \delta \bar{\nu}_{\mu R}, \quad (5)$$

$$i \nu'_{\mu L} = s_2 \delta \nu_{eL} - \mu B_\perp e^{i\phi} \bar{\nu}_{eR} + (V_\mu + c_2 \delta) \nu_{\mu L}, \quad (6)$$

$$i \bar{\nu}'_{\mu R} = \mu B_\perp e^{-i\phi} \nu_{eL} + s_2 \delta \bar{\nu}_{eR} - (V_\mu - c_2 \delta) \bar{\nu}_{\mu R}. \quad (7)$$

Here  $V_e = \sqrt{2}G_F[N_e(r) - N_n(r)/2]$  and  $V_\mu = \sqrt{2}G_F[-N_n(r)/2]$  are the matter-induced potentials of electron and muon neutrinos, respectively,  $N_e$  and  $N_n$  being the electron and neutron number densities;  $s_2 = \sin 2\theta$ ,  $c_2 = \cos 2\theta$ ,  $\delta = \Delta m^2/4E$ . In eqs. (4)-(7) the angle  $\phi$  defines the direction of the magnetic field  $\mathbf{B}_\perp(r)$  in the plane orthogonal to the neutrino momentum,  $B_\perp(r) = |\mathbf{B}_\perp(r)|$ . In the case of twisting magnetic fields ( $\phi = \phi(r)$ ), the probability of  $\bar{\nu}_{eR}$  production inside the sun can be enhanced [21, 22]; since little is known about the possible twist of the solar magnetic field, we set  $\phi = 0$  hereafter. This approximation is expected to reduce the calculated flux of solar  $\bar{\nu}_{eR}$ 's and therefore make our upper bounds on  $\mu B_\perp$  more conservative.

Since we are interested in the situation when the SFP probability is small, it can be calculated in perturbation theory. To find the  $\bar{\nu}_{eR}$  amplitude in leading order in  $\mu B_\perp$ , one needs to use in eq. (7) the “unperturbed” by SFP value of the amplitude  $\nu_{eL}$ . This means that the  $\mu B_\perp$  terms in eqs. (4) and (6) should be omitted. These two equations then decouple from the rest of the system and reduce to the standard two-flavour evolution equations describing neutrino oscillations in matter. Next, we take into account that, due to the partial cancellation between the amplitudes of transitions (1) and (2) discussed above, one can neglect the  $\bar{\nu}_{eR}$  amplitude inside the sun; we therefore omit eq. (5) as well as the  $\bar{\nu}_{eR}$  term in eq. (7). From eq. (7) one then obtains, up to an irrelevant phase factor, the following expression for the amplitude of the  $\nu_{eL} \rightarrow \bar{\nu}_{\mu R}$  transition:

$$A(\nu_{eL} \rightarrow \bar{\nu}_{\mu R} R_\odot) = \int_{r_i}^{R_\odot} \mu B_\perp(r) \nu_{eL}(r) e^{-i \int_{r_i}^r (V_\mu - c_2 \delta) dr'} dr. \quad (8)$$

Here  $r_i$  is the coordinate of the point at which the  $\nu_{eL}$  was produced in the sun, and the amplitude  $\nu_{eL}(r)$  is determined solely by neutrino oscillations, i.e. has to be found from eqs. (4) and (6) with  $\mu B_\perp = 0$ .

It is easy to check that for the LMA and LOW solutions of the solar neutrino problem the solar  $\nu_{eL} \leftrightarrow \nu_{\mu L}$  oscillations are adiabatic; one therefore can use the adiabatic approximation, which yields

$$\nu_{eL}(r) = \cos \theta(r_i) \cos \theta(r) e^{-i \int_{r_i}^r E_1 dr'} + \sin \theta(r_i) \sin \theta(r) e^{-i \int_{r_i}^r E_2 dr'} . \quad (9)$$

Here

$$E_{1,2}(r) = \frac{V_e + V_\mu}{2} \mp \omega, \quad \omega = \sqrt{\left( \frac{V_e - V_\mu}{2} - c_2 \delta \right)^2 + (s_2 \delta)^2}, \quad (10)$$

and  $\theta(r)$  is the mixing angle in matter at a point  $r$ , defined through

$$\cos 2\theta(r) = \frac{c_2 \delta - \frac{V_e - V_\mu}{2}}{\omega}. \quad (11)$$

Substituting (9) into (8) one finds

$$A(\nu_{eL} \rightarrow \bar{\nu}_{\mu R} R_\odot) = \int_{r_i}^{R_\odot} \mu B_\perp(r) [\cos \theta(r_i) \cos \theta(r) e^{-ig_1(r)} + \sin \theta(r_i) \sin \theta(r) e^{-ig_2(r)}] dr, \quad (12)$$

where

$$g_{1,2}(r) = \int_{r_i}^r \left( \frac{V_e + 3V_\mu}{2} - c_2 \delta \mp \omega \right) dr'. \quad (13)$$

This expression is relevant for the LMA and LOW solutions of the solar neutrino problem.

In the case of the VO solution, neutrino oscillations inside the sun are essentially blocked, so that instead of eq. (9) one finds from eq. (4)

$$\nu_{eL}(r) = e^{-i \int_{r_i}^r (V_e - c_2 \delta) dr'}. \quad (14)$$

Eq. (8) then yields

$$A(\nu_{eL} \rightarrow \bar{\nu}_{\mu R} R_\odot) = \int_{r_i}^{R_\odot} \mu B_\perp(r) e^{-i \int_{r_i}^r (V_e + V_\mu - 2c_2 \delta) dr'} dr. \quad (15)$$

### 3 Calculations of the expected $\bar{\nu}_{eR}$ flux

Since only the solar  $^8\text{B}$  neutrinos with the energies  $E > 5$  MeV can contribute to an observable flux of solar antineutrinos, the expected flux of  $\bar{\nu}_{eR}$ 's in this energy region is  $\Phi_{\bar{\nu}_e}(E) = \Phi_{sB}(E) P(\nu_{eL} \rightarrow \bar{\nu}_{eR}, E)$ . We shall now concentrate on the calculation of the transition probability  $P(\nu_{eL} \rightarrow \bar{\nu}_{eR}, E)$  for the LMA, LOW and VO solutions of the solar neutrino problem.

In eqs. (12) and (15), the pre-exponential factors in the integrands are in general smooth functions of  $r$ , but the complex exponential factors are very rapidly oscillating functions. The oscillations are especially fast for the LMA solution because it corresponds to the largest values of  $\Delta m^2$ . The integrals of this type are notoriously difficult to calculate numerically – one needs very fine integration steps in order for the integrals to converge. For example, for the LMA case a step  $\sim 10^{-6} R_\odot$  or smaller is necessary.

Fortunately, there are well developed and accurate approximate analytic methods of calculating such integrals (see, e.g., [27]). For our purposes, analytic expressions also have an advantage of allowing one to directly relate the expected flux of solar  $\bar{\nu}_{eR}$  to simple characteristics of the solar magnetic field. The integrals in eqs. (12) and (15) are of general type

$$I = \int_a^b f(x) e^{-ig(x)} dx, \quad (16)$$

where  $f(x)$  is a smooth function of  $x$  and  $|g'(x)|$  is large except possibly in the vicinity of a finite number of points in the interval  $(a, b)$ . The integrals of rapidly oscillating functions are in general strongly suppressed because the contributions of neighbouring points tend to cancel each other. The exceptions are the endpoints of the integration interval, for which there are no neighbouring points on one of the sides, and the points where  $g'(x) = 0$ , which correspond to the extrema of the phase  $g(x)$ . In the vicinity of these (stationary phase) points the phase changes slowly and the corresponding contributions to the integral are not suppressed. These contributions can be found in the stationary phase approximation, which is the complex version of the steepest descent approximation.

The contributions of the stationary phase points to the integrals of the type (16) are  $\mathcal{O}(1/\sqrt{|g''|})$ , whereas the endpoint contributions are in general  $\mathcal{O}(1/|g'|)$ . It is easy to check that for the integrals in eqs. (12) and (15) and for the values of neutrino parameters relevant for the LMA, LOW and VO solution of the solar neutrino problem the condition

$$|g''(x)|/g'(x)^2 \ll 1 \quad (17)$$

is always satisfied. Therefore, if there are stationary phase points in the integration interval, they are in general expected to give the dominant contributions to the integrals. However, as we shall show now, this does not happen in the cases of interest to us because the stationary points either do not exist or their contributions are strongly suppressed by nearly vanishing pre-exponential factors in (16).

Let us check the stationary phase conditions for various solutions of the solar neutrino problem. If one disregards neutrino oscillations and considers only SFP, the amplitude of the  $\nu_{eL} \rightarrow \bar{\nu}_{\mu R}$  transitions is given by eq. (15). The stationary phase condition  $g' = 0$  then reduces to

$$V_e + V_\mu - 2c_2\delta = 0. \quad (18)$$

This is nothing but the resonance condition for neutrino SFP in the small  $\theta$  limit [7, 8]. As was discussed above, neutrino oscillations inside the sun can only be neglected in the case

of the VO solution. Since for this solution the parameter  $\delta$  is very small ( $\sim 10^{-17}$  eV), the resonance condition (18) is satisfied essentially at the surface of the sun, where the magnetic field strength is known to be very small ( $\sim 10 - 100$  G); therefore the stationary phase point plays no role in this case.

For the LMA and LOW solutions, the stationary phase conditions  $g'_{1,2} = 0$  reduce to

$$V_e + V_\mu - 2c_2\delta = \frac{(s_2\delta)^2}{2V_\mu}, \quad (19)$$

which is the SFP resonance condition in the presence of mixing. It is easy to see that this condition cannot be satisfied for large values of mixing angles  $\theta$ . Indeed, eq. (19) has solutions only when

$$\sin^2 2\theta \leq \frac{1 - Y_e}{Y_e}, \quad (20)$$

where  $Y_e$  is the number of electrons per nucleon in matter. Its value varies between 0.667 and 0.868 inside the sun, so that condition (20) requires  $\sin^2 2\theta < 0.5$ , too small for both LMA and LOW solutions. Thus, there are no stationary phase points contributions to the amplitude of the  $\nu_{eL} \rightarrow \bar{\nu}_{\mu R}$  transition in these cases as well, and one has to consider the endpoint contributions.

To obtain the contributions of the endpoints of the integration interval to an integral of the type (16) we integrate it by parts. Integrating two times one finds

$$\int_a^b f(x) e^{-ig(x)} dx = \left[ \left( i \frac{f(x)}{g'(x)} + \frac{f'(x)}{g'(x)^2} - \frac{f(x)g''(x)}{g'(x)^3} \right) e^{-ig(x)} \right] \Big|_a^b + \mathcal{O}(1/g'(x)^3). \quad (21)$$

From (17) it follows that the third term in the brackets can always be neglected. We shall now consider the amplitudes of the  $\nu_{eL} \rightarrow \bar{\nu}_{eR}$  transition for all three solutions of the solar neutrino problem of interest.

### 3.1 LMA solution

The current best fit values of the neutrino parameters for this solution are  $\Delta m^2 \simeq 6 \times 10^{-5}$  eV<sup>2</sup>,  $\sin 2\theta \simeq 0.9$  [23]. We will be interested in neutrino energies  $E \simeq (5 - 15)$  MeV, so that  $\delta \simeq (1 - 3) \times 10^{-12}$  eV. It is easy to check that the contribution of the first term in the square brackets in eq. (12) is always at least one order of magnitude smaller than that of the second term, and so we shall neglect it. We now apply formula (21). One has  $f' \propto (d/dr)[\sin \theta B_\perp] \simeq \sin \theta B'_\perp$  since in the adiabatic regime  $\theta'$  is small. The comparison of the first and the second terms in the brackets in eq. (21) then shows that the first term dominates when

$$B_\perp(r_i) \gg |B'_\perp(r_i)|/g'_2(r_i) \simeq 10^{-4} R_\odot |B'_\perp(r_i)|. \quad (22)$$

We consider only the contribution of the neutrino production point  $r_i$  since the magnetic field at the final point of evolution  $r = R_\odot$  is negligible. Let us introduce the scale height

for the solar magnetic field strength  $L_B = |B_\perp^{-1}(dB_\perp(r)/dr)|^{-1}$ , which is a characteristic distance over which the magnetic field varies significantly. Condition (22) then can be written as

$$L_B(r_i) \gg 10^{-4} R_\odot. \quad (23)$$

Magnetic fields of scale heights as small as  $L_B \lesssim 10^{-4} R_\odot$  can only exist over very short distances and so cannot lead to any sizeable SFP. We shall therefore consider large-scale solar magnetic fields which satisfy (23).

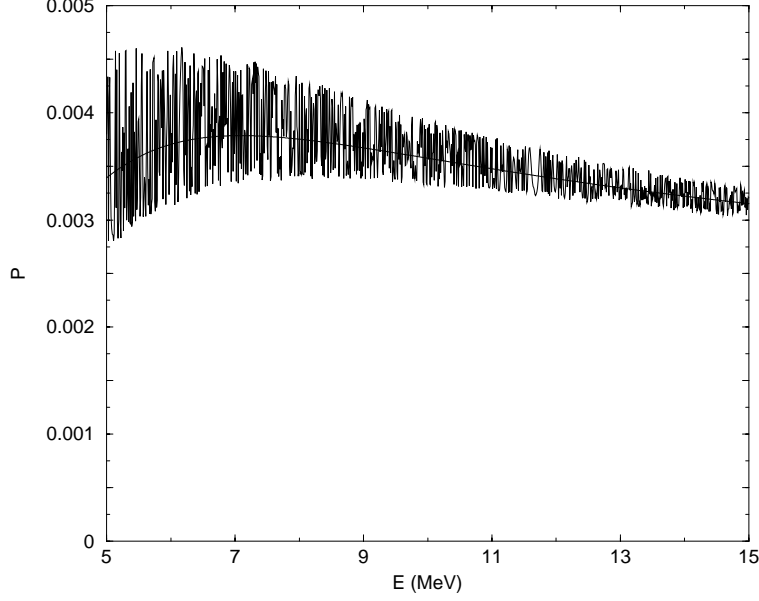


Figure 1: Probabilities  $P(\nu_{eL} \rightarrow \bar{\nu}_{eR})$  corresponding to the  $\nu_{eL} \rightarrow \bar{\nu}_{\mu R}$  amplitudes of eqs. (12) (wiggly curve) and (24) (smooth curve) for the LMA solution. Magnetic field linearly decreasing from  $B_0 = 5 \times 10^7$  G at  $r = 0.05R_\odot$  to zero at  $r = R_\odot$  and  $\mu = 10^{-12} \mu_B$  were chosen.

The maximum of production of solar  $^8\text{B}$  neutrinos which are of interest to us corresponds to  $r_0 \simeq 0.05R_\odot$ ; from eqs. (21) and (12) we then obtain

$$A(\nu_{eL} \rightarrow \bar{\nu}_{\mu R} R_\odot) \simeq \left[ \frac{\sin^2 \theta(r_i) \mu B_\perp(r_i)}{g'_2(r_i)} \right] \Big|_{r_i=0.05R_\odot}, \quad (24)$$

where we once again omitted an irrelevant phase factor. In the energy region of interest, the corresponding probability varies by less than 20% (see fig. 1), and so in first approximation we can replace it by its mean value. From eqs. (3), (24) and (13) one then finds

$$P(\nu_{eL} \rightarrow \bar{\nu}_{eR}) \simeq 1.8 \times 10^{-10} \sin^2 2\theta \left[ \frac{\mu}{10^{-12} \mu_B} \frac{B_\perp(0.05R_\odot)}{10 \text{ kG}} \right]^2, \quad (25)$$

where we have taken into account that the  $\bar{\nu}_{\mu R} \rightarrow \bar{\nu}_{eR}$  oscillations in the space between the sun and the earth are in the averaging regime. Eq. (25) is our final result for the LMA case.



In fig. 1. the probabilities  $P(\nu_{eL} \rightarrow \bar{\nu}_{eR})$  corresponding to the  $\nu_{eL} \rightarrow \bar{\nu}_{\mu R}$  amplitudes of eqs. (12) and (24) are shown (the wiggly and smooth curves, respectively). The fast oscillations of the former are due to the interference between the two terms in eq. (12), whereas the latter is smooth because in obtaining it we neglected the (subleading) first term in (12). Notice that the oscillations described by the wiggly curve are in fact unobservable since they average out when one integrates over the neutrino production region or takes into account finite energy resolution of neutrino detectors.

### 3.2 LOW solution

The best fit values of the neutrino parameters for this solution are  $\Delta m^2 \simeq 10^{-7} \text{ eV}^2$ ,  $\sin 2\theta \simeq 0.98$  [23]. For the interval of neutrino energies of interest, the parameter  $\delta$  is in the range  $\delta \simeq (1.7 - 5) \times 10^{-15} \text{ eV}$ . Once again, the contribution of the first term in the square brackets in eq. (12) is much smaller than that of the second term, and so we neglect it. The analysis is similar to that in the LMA case, but there is one important difference. In the LMA case, the value of  $g'_2$  changes only by about a factor of two from the neutrino production point to the surface of the sun ( $g'_2 R_\odot$  varies between  $\sim 10^4$  and  $\sim 5 \times 10^3$ ). In contrast to this, in the LOW case it changes by a large factor – from  $g'_2 R_\odot \sim 10^4$  at  $r \simeq 0.05 R_\odot$  to  $g'_2 R_\odot \sim 10$  at  $r = R_\odot$ . This difference in the behaviour of  $g'_2$  is a consequence of the fact that the values of  $\delta$  are much larger and so contribute significantly to  $g'_2$  at all values of  $r$  in the LMA case whereas in the LOW case they dominate near the surface of the sun but are negligible in the solar core; the values of  $g'_2$  at small  $r$  are mainly determined by the matter-induced potentials. Indeed, in the LOW case one finds from eqs. (13) and (10)

$$g'_2|_{r=R_\odot} \simeq (1 - c_2) \delta, \quad g'_2|_{r=0.05 R_\odot} \simeq (V_e + V_\mu)|_{r=0.05 R_\odot}. \quad (26)$$

The smallness of  $g'_2$  at  $r = R_\odot$  implies that the integral in eq. (12) is now dominated by the surface of the sun. Since the magnetic field nearly vanishes there, one can expect the main contribution to the integral to come from the second term in brackets in eq. (21), i.e. from the term proportional to the derivative of the solar magnetic field rather than to the field itself. Indeed, the condition for the domination of the term  $\propto B'_\perp$  is

$$B_\perp(R_\odot) \ll [|B'_\perp(r)|/g'_2(r)]|_{r=R_\odot}, \quad \text{or} \quad L_B(R_\odot) \ll 0.1 R_\odot. \quad (27)$$

Since the magnetic field at the surface of the sun is very weak, this condition is likely to be satisfied. For example, it is satisfied for any magnetic field profile which near the surface of the sun has the form  $B_\perp = B_0[1 - (r/R_\odot)^n] + B_1$ , provided that  $n > 10(B_1/B_0)$ ; for the magnetic field at the solar surface  $B_1 = 100 \text{ G}$  and the peak value of the field  $B_0 = 10 \text{ kG}$  this requires  $n > 0.1$ . In what follows we shall be assuming that condition (27) is satisfied.

The amplitude of the  $\nu_{eL} \rightarrow \bar{\nu}_{\mu R}$  transition is then approximately given by

$$A(\nu_{eL} \rightarrow \bar{\nu}_{\mu R} \ R_\odot) \simeq \sin \theta(r_i) \sin \theta \frac{\mu |B'_\perp(r)|_{r=R_\odot}}{(1 - c_2)^2 \delta^2}. \quad (28)$$

We shall parameterize the derivative of the magnetic field strength at  $r = R_\odot$  as

$$|B_\perp(r)'|_{r=R_\odot} = \frac{B_0}{0.15 R_\odot} \kappa, \quad (29)$$

with  $B_0$  the peak value of the field of the convective zone ( $0.7R_\odot \leq r \leq R_\odot$ ), and  $\kappa$  a parameter;  $\kappa = 1$  would correspond to, e.g., a linearly decreasing towards the surface of the sun field with the maximum at the center of the convective zone. The probability of the  $\nu_{eL} \rightarrow \bar{\nu}_{eR}$  transition is then

$$P(\nu_{eL} \rightarrow \bar{\nu}_{eR}) \simeq 1.85 \times 10^{-5} \frac{\cos^2 \theta}{\sin^4 \theta} \kappa^2 \left( \frac{3 \times 10^{-15} \text{ eV}}{\delta} \right)^4 \left[ \frac{\mu}{10^{-12} \mu_B} \frac{B_0}{10 \text{ kG}} \right]^2, \quad (30)$$

where we have taken into account that the  $\bar{\nu}_{\mu R} \rightarrow \bar{\nu}_{eR}$  oscillations in the space between the sun and the earth are in the averaging regime, and that the mixing angle in matter at the neutrino production point  $\theta(r_i) \simeq \pi/2$ . As an example, for the best fit values of neutrino parameters for the LOW solution,  $E = 10 \text{ MeV}$ ,  $\mu = 10^{-12} \mu_B$ ,  $B_0 = 100 \text{ kG}$  and  $\kappa = 1$ , eq. (30) yields  $P(\nu_{eL} \rightarrow \bar{\nu}_{eR}) \simeq 1.4 \times 10^{-2}$ .

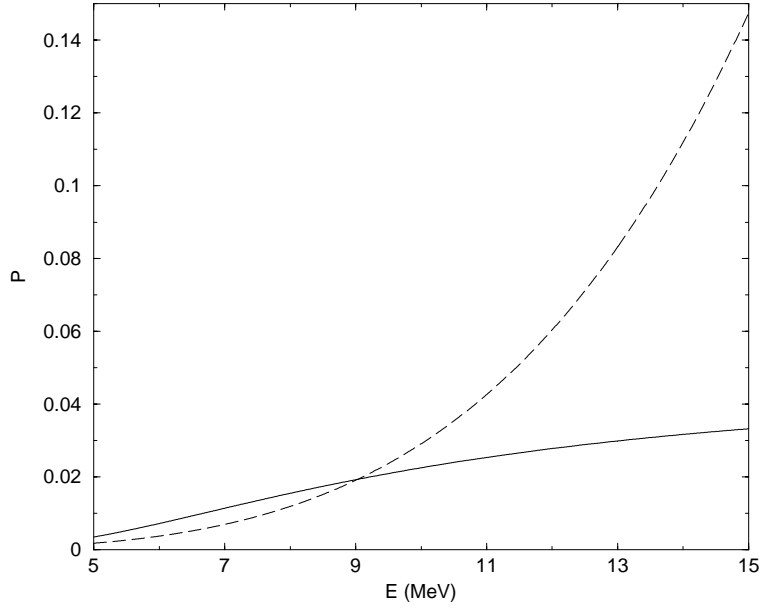


Figure 2: Probabilities  $P(\nu_{eL} \rightarrow \bar{\nu}_{eR})$  corresponding to the  $\nu_{eL} \rightarrow \bar{\nu}_{\mu R}$  amplitudes of eqs. (12) (solid curve) and (28) (dashed curve) for the LOW solution. Magnetic field decreases linearly from  $B_0 = 10^6 \text{ G}$  at  $r = 0.05R_\odot$  to zero at  $r = R_\odot$ ,  $\mu = 10^{-12} \mu_B$ .

It should be noted that, while the probability  $P(\nu_{eL} \rightarrow \bar{\nu}_{eR})$  based on the perturbation-theoretic  $\nu_{eL} \rightarrow \bar{\nu}_{\mu R}$  amplitude of eq. (12) (fig. 2, solid curve) is very accurate, the simplified expression (30) (dashed curve in fig. 2) is only correct within a factor of three or four. The

reason for that is the following. In the case of the LOW solution the quantity  $g'_2 R_\odot$  changes only by about a factor of two throughout the solar convective zone, being  $\sim 10 - 20$ , i.e. not too large. Therefore the contribution of the integration endpoint at the solar surface to the integral in (12) is not much bigger than that of the convective zone, and the approximation (30) is only an order of magnitude estimate.

We pointed out earlier that the production of  $\bar{\nu}_{eR}$  inside the sun is suppressed because of the partial cancellation between the amplitudes of the channels (1) and (2). This cancellation is, however, partly compensated in the case of the LOW solution by an enhancement due to the fact that  $c_2 \delta$  is small and so is  $V_e$  in the solar convective zone, which is the region where the SFP mainly occurs. Therefore in the case of the LOW solution there is an additional channel of the  $\bar{\nu}_{eR}$  production – direct production inside the sun through neutrino oscillations and SFP. At the same time, the flux of  $\bar{\nu}_{eR}$ 's generated via the mechanism that we discussed so far is somewhat reduced if one takes the direct production into account; as a result, the total  $\bar{\nu}_{eR}$  flux changes only slightly. We have checked by a numerical integration of the system of the differential equations (1)-(4) that our approximation, in which we neglect the direct production but consider unsuppressed production through the chain of the processes (2) yields a very accurate prediction of the total solar  $\bar{\nu}_{eR}$  flux.

### 3.3 VO solution

The best fit values of the neutrino parameters for this solution are  $\Delta m^2 \simeq 4.5 \times 10^{-10} \text{ eV}^2$ ,  $\sin 2\theta \simeq 0.93$  [23]. The  $\nu_{eL} \rightarrow \bar{\nu}_{\mu R}$  transition amplitude is given by eq. (15). For the interval of neutrino energies of interest, the parameter  $\delta$  is in the range  $\delta \simeq (0.77 - 2.3) \times 10^{-17} \text{ eV}$ , i.e. is negligible compared to  $V_e + V_\mu$  essentially everywhere in the sun. One can therefore put  $\delta = 0$  in eq. (15), i.e. the  $\nu_{eL} \rightarrow \bar{\nu}_{\mu R}$  transition probability is practically energy independent.

In the case of the VO solution, the parameter  $g'$  that plays a key role for integrals of rapidly oscillating functions of the type (16) is simply  $g' = V_e + V_\mu$ . The quantity  $g'(r)R_\odot$  changes from a value  $\sim 10^4$  in the center of the sun to nearly zero at its surface, being  $\lesssim 1$  inside the convective zone. Thus, it is not legitimate to use the approximate expressions of the type (21) which require  $g'R_\odot$  to be very large everywhere in the integration interval. We shall therefore estimate the transition amplitude differently in this case.

Let us first notice that the phase  $g(r)$  that corresponds to eq. (15),

$$g(r) = \int_0^r (V_e + V_\mu) dr', \quad (31)$$

first grows rapidly with  $r$  and then saturates and slowly approaches its asymptotic value because of the steep decrease of  $V_e + V_\mu$ . The contribution to the integral (15) from the region where  $g(r)$  rapidly grows is strongly suppressed, and the main contribution comes from the region where the phase changes little. We can therefore adopt an approximation of retaining only the contribution of the region  $r > r_0$  where the change of the phase  $\Delta g \lesssim 1$

and neglecting the phase change in this interval. Since it is the change of the phase and not its absolute value that matters for the transition probability, this amounts to merely replacing in this interval the integrand of eq. (15) by  $\mu B_{\perp}(r)$ . The lower boundary of the new integration interval  $r_0$  is defined from the condition

$$g(R_{\odot}) - g(r_0) \simeq 1. \quad (32)$$

As a result, we obtain

$$P(\nu_{eL} \rightarrow \bar{\nu}_{eR}) \simeq \left| \mu \int_{r_0}^{R_{\odot}} B_{\perp}(r) dr \right|^2 \sin^2 2\theta \sin^2 \left( \frac{\Delta m^2}{4E} R_{es} \right). \quad (33)$$

Solving eq. (32) for  $r_0$  we find  $r_0 \simeq 0.817R_{\odot}$ . We compared the  $\nu_{eL} \rightarrow \bar{\nu}_{eR}$  probabilities obtained using this approximation with those based on the full  $\nu_{eL} \rightarrow \bar{\nu}_{\mu R}$  amplitude (15) for a number of magnetic field profiles and found that the accuracy of the approximation (33) is typically about 20%.

Let us introduce the average magnetic field strength in the interval  $r_0 \leq r \leq R_{\odot}$  through

$$\overline{B}_{\perp} = \frac{1}{R_{\odot} - r_0} \int_{r_0}^{R_{\odot}} B_{\perp}(r) dr, \quad (34)$$

where  $R_{\odot} - r_0 = 0.183R_{\odot}$ . Then eq. (33) can be rewritten as

$$P(\nu_{eL} \rightarrow \bar{\nu}_{eR}) \simeq 1.4 \times 10^{-3} \left[ \frac{\mu}{10^{-12} \mu_B} \frac{\overline{B}_{\perp}}{10 \text{ kG}} \right]^2 \sin^2 2\theta \sin^2 \left( \frac{\Delta m^2}{4E} R_{es} \right). \quad (35)$$

## 4 Discussion

We calculated the probability of production of solar  $\bar{\nu}_{eR}$ 's assuming that the solar neutrino deficit is due to neutrino oscillations while the spin-flavour precession caused by the interaction of neutrino transition magnetic moments with the solar magnetic field is present as a subdominant process. We considered the SFP in perturbation theory and obtained analytic expressions for the transition probability  $P(\nu_{eL} \rightarrow \bar{\nu}_{eR})$  valid for the LMA, LOW and VO solutions of the solar neutrino problem. We compared these analytical expressions with the results of numerical integration of the system of differential equations (1)-(4) and found very good agreement in all the cases.

For each of the solutions of the solar neutrino problem we then obtained simplified approximate expressions for  $P(\nu_{eL} \rightarrow \bar{\nu}_{eR})$ , which allowed us to relate this probability with simple characteristics of the solar magnetic field  $B_{\perp}(r)$ . For different solutions, different characteristics of the solar magnetic field are probed: for the VO solution, the  $\bar{\nu}_e$  flux is determined by the integral of  $B_{\perp}(r)$  over the upper 2/3 of the solar convective zone, for the LOW solution it depends on the derivative of  $B_{\perp}(r)$  at the surface of the sun, and for LMA it is determined by the magnitude of  $B_{\perp}$  in the neutrino production region.

The accuracy of the simplified expressions for  $P(\nu_{eL} \rightarrow \bar{\nu}_{eR})$  is also different for different solutions: the error is less than 3% for the LMA solution and about 20% for the VO solution, while for the LOW solution the simplified expression is only correct within a factor of three or four. Since the efficiency of SFP depends on the product of neutrino magnetic moment and magnetic field strength, only this product and not  $\mu$  and  $B_\perp$  separately can be probed by studying the solar  $\bar{\nu}_e$  flux  $\Phi_{\bar{\nu}_e}$ . Comparing eqs. (25), (30) and (35) we find that the sensitivity of the  $\bar{\nu}_e$  flux to the product  $\mu B_\perp$  is strongest in the case of the VO solution of the solar neutrino problem; it is weaker in the case of the LOW solution and weakest for the LMA solution. This, however, does not necessarily mean that the LMA solution has the lowest sensitivity to the neutrino magnetic moment: the  $\bar{\nu}_{eR}$  flux in that case depends on the magnetic field in the core of the sun which may well be much stronger than the field in the convective zone, relevant for the VO case.

We shall now discuss the present experimental upper bounds on  $\Phi_{\bar{\nu}_e}$  as well as the sensitivity of the future experiments, and their implications. Currently, the most stringent upper bounds on  $\Phi_{\bar{\nu}_e}$  come from the LSD experiment,  $\Phi_{\bar{\nu}_e} < (1.7 \times 10^{-2}) \Phi_{sB}$  at 90% CL [28], and from the Super-Kamiokande experiment,  $\Phi_{\bar{\nu}_e} < (1.2 - 1.6) \times 10^{-2} \Phi_{sB}$  at 90% CL [29]. The Super-Kamiokande bounds were presented for several energy bins with  $E \geq 8$  MeV. Future experiments are expected to improve these bounds (or discover the flux of solar  $\bar{\nu}_{eR}$ ): KamLAND will be able to put a limit of  $10^{-3} \Phi_{sB}$  at 95% CL on the solar  $\bar{\nu}_{eR}$  flux after one year of operation [30], and Borexino should be able to reach a similar sensitivity after a few years of data taking [31]. The current limit  $\Phi_{\bar{\nu}_e} \lesssim 1.5\% \Phi_{sB}$  implies, in the case of the LMA solution, a bound

$$\left[ \frac{\mu}{10^{-12} \mu_B} \frac{B_\perp(r_i)}{10 \text{ kG}} \right] \lesssim 10^4, \quad (36)$$

where  $B_\perp(r_i)$  is the average solar magnetic field in the neutrino production region,  $r \lesssim 0.1 R_\odot$ . An experiment with the tritium radioactive antineutrino source has been recently proposed with the goal of putting an upper limit of  $3 \times 10^{-12} \mu_B$  on the neutrino magnetic moment  $\mu$  or measuring it if it is above this value [32]; if  $\mu \simeq 3 \times 10^{-12} \mu_B$  is found, the bound (36) would imply  $B_\perp(r_i) \lesssim 3 \times 10^7$  G. Note that this limit is more stringent than the astrophysical one obtained from the requirement that the pressure of the solar magnetic field should not exceed the matter pressure ( $B_\perp \lesssim 10^9$ ) [33].

Conversely, if a reliable quantitative model of the solar magnetic field is developed, eq. (36) will limit the neutrino magnetic moment. For  $B_\perp(r_i)$  close to the above-mentioned astrophysical bound, the limit on  $\mu$  would be  $\mu \lesssim 10^{-13} \mu_B$ , which is more than an order of magnitude more stringent than the expected limit from the planned laboratory experiment [32]. Unfortunately, no compelling model of the solar magnetic field exists at present.

Similar considerations apply to the LOW and VO cases. For LOW, the current limits on the solar  $\bar{\nu}_{eR}$  flux lead to

$$\kappa \left[ \frac{\mu}{10^{-12} \mu_B} \frac{B_0}{10 \text{ kG}} \right] \lesssim 10, \quad (37)$$

where  $\kappa$  and  $B_0$  parameterize the derivative of the solar magnetic field at  $r = R_\odot$ , see

eq. (29). For  $\mu \simeq 3 \times 10^{-12} \mu_B$ , eq. (37) limits this derivative to be  $|B'_\perp(r)|_{R_\odot} \lesssim 2.2 \times 10^2$  kG/ $R_\odot$ . Conversely, if the actual value of  $|B'_\perp(r)|_{R_\odot}$  is close to this value, the limit on  $\mu$  from (37) would be competitive with the expected upper bound from the planned laboratory experiment [32].

In the VO case, the current upper bounds on  $\Phi_{\bar{\nu}_e}$  imply

$$\left[ \frac{\mu}{10^{-12} \mu_B} \frac{\bar{B}}{10 \text{ kG}} \right] \lesssim 5, \quad (38)$$

where  $\bar{B}$  is the average magnetic field in the interval  $0.817 R_\odot \leq r \leq R_\odot$  defined in (34). For  $\mu = 3 \times 10^{-12}$  this gives  $\bar{B} < 17$  kG. If, alternatively, some model considerations establish that the average field  $\bar{B}$  is, for example, 100 kG, eq. (38) would lead to the limit  $\mu < 5 \times 10^{-13}$ .

With the expected upper bound on the flux of solar  $\bar{\nu}_{eR}$  from KamLAND, all the limits that we discussed above (eqs. (36) – (38)) will be strengthened by about a factor of four.

In the VO case, the flux of solar  $\bar{\nu}_{eR}$ 's will have seasonal variations, similar to those of the  $\nu_{eL}$  flux. In the LOW and VO cases, for which the  $\bar{\nu}_{eR}$  production is mainly driven by the magnetic field in the convective zone, the solar  $\bar{\nu}_{eR}$  flux may also vary with time due to the 11-year variations of this magnetic field. For all the solutions, the solar  $\bar{\nu}_{eR}$  flux should, of course, also exhibit  $\sim 7\%$  variations due to the variations of the distance between the sun and the earth. Low statistics may, however, make these variations difficult to detect.

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